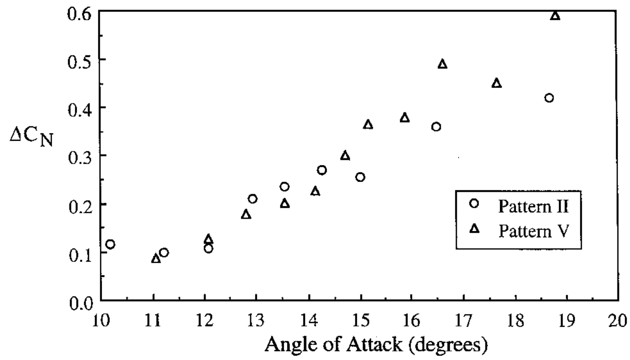

Fig. 3 Incremental normal force coefficient at $C_q = 5.0 \times 10^{-4}$ for suction patterns II and V.

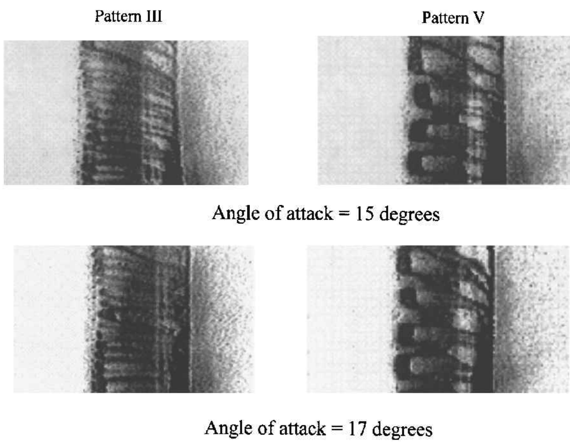

Fig. 4 Comparison of the surface flows of patterns III and V at $C_q = 5.0 \times 10^{-4}$.

sensitive to the suction surface geometry for a sufficiently high suction rate. This indicates that the broadband nature of the instability is retained.

III. Conclusion

Several conclusions can be drawn from the study.

1) A periodical surface flow control can produce a three-dimensional perturbation that leads to the formation of streamwise vortical flow structures in the separated shear layer. Though suction was used in this study, boundary-layer controls in general should achieve the same effect when applied in a similar fashion.

2) The vortical structures cause a normally separated flow to reattach.

3) The control is most effective when applied near and upstream of the natural separation.

4) The control device size, as measured in this particular case by the combination of suction hole diameter, hole number density, and associated porosity, is small.

5) Similar results are obtained over different control patterns, indicating a broadband instability.

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Planar Symmetry in the Unsteady Wake of a Sphere

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Introduction

THE sphere can be considered a prototypical axisymmetric bluff body, and understanding the sphere wake is important because of its relevance to a number of aerodynamic and hydrodynamic applications. In addition, vortex shedding from particles assumes importance in particulate flows at high-particle Reynolds numbers because this shedding can have a significant effect on the enhancement of turbulence.[1] In most cases particles can be modeled as spheres; therefore, an in-depth study of the vortex dynamics in sphere wakes would greatly improve our understanding of particle-turbulence interaction.

The sphere wake is nowhere as well understood as its two-dimensional counterpart, the circular cylinder, which has been the focus of intense experimental and numerical investigations in the last three decades.[2] The relatively few studies of the sphere wake that have been done to date show that the vortex topology and shedding process is significantly different from that in the cylinder wake.[3–8] Our understanding of cylinder wakes is, therefore, not directly useful in interpreting our observations of sphere wakes. In particular, there are a number of states in the steady and unsteady sphere wake that do not have a counterpart in two-dimensional wakes. These include the nonaxisymmetric steady state (or double-thread wake), which occurs at $210 < Re_d < 270$ (where Re_d is the Reynolds number based on the diameter and freestream velocity), and the presence of planar symmetry in the unsteady wake regime at higher Reynolds numbers. From experiments,[3–6] stability analysis,[8] and numerical simulations,[7] it is known that the sphere wake becomes unsteady at a Reynolds number of about 270. The unsteadiness first appears as a waviness in the double-thread wake. As the Reynolds number is increased to about 290, the wavy motion gives way to vortex shedding wherein streamwise-oriented vortex loops are formed in the wake at periodic intervals.[3–6] A unique feature of the wake in this regime is that it seems to exhibit symmetry about a plane passing through the wake centerline. The presence of planar symmetry in the unsteady sphere wake is in fact peculiar enough that it is viewed by some with a degree of skepticism. This skepticism is encouraged further because of the lack of a detailed analysis of this flow regime.

The objective of the current Note is therefore to describe the salient features of this vortex shedding regime. In particular, in this Note we will 1) clearly and unequivocably establish that such a flow regime does exist, 2) describe the structure of the vortices and resultant forces on the cylinder in this regime, and 3) provide a more accurate estimate of the upper extent of this regime and an understanding of the bifurcation through which planar symmetry is lost at higher Reynolds numbers. All of these issues are addressed here through direct numerical simulations of the sphere wake.

An accurate Fourier–Chebyshev spectral collocation method has been used for computing three-dimensional, unsteady, viscous incompressible flow past a sphere. The solution is advanced in time by using a second-order-accurate, two-step, time-split scheme. The current solver has been tested extensively by performing a series of simulations of flow past a sphere in the Reynolds number range $50 < Re_d < 500$ and comparing our numerical results with established experimental data.[9] In addition to qualitative features, key quantities such as drag coefficients, separation angles, and vortex

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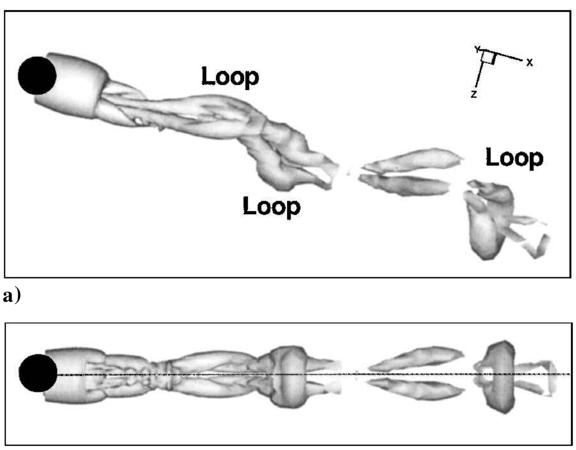
shedding Strouhal numbers show good agreement with experimental data throughout this Reynolds number range. Furthermore, extensive tests have been carried out to demonstrate spectral accuracy and adequacy of the nonreflective outflow boundary condition. A detailed description of the solver and the validation study can be found in Ref. 9.

Results

Here we will focus first on describing our simulation results for $Re_d = 350$, which lies in the middle of the range where planar symmetry is indicated by experiments.[4] In this simulation a mean axisymmetric flow is obtained first. Subsequently, a small disturbance in the form of a random (white noise) azimuthal slip velocity is provided for a short time. The disturbance grows in time because of the inherent instability of the flow and eventually saturates. The simulation is continued well beyond the time where a stationary flow is obtained, and all results presented here are for this stationary flow regime. It is important to reemphasize that the perturbation provided is random in space and therefore does not have a preferred spatial structure or orientation.

Figure 1 shows two views of the vortex topology observed for $Re_d = 350$ at one time instant. The vortical structures in these and subsequent figures are identified by the imaginary part of the complex eigenvalue of the velocity gradient tensor.[10,11] Figure 1a shows the three-dimensional structure of the wake, and vortex shedding is characterized by the appearance of interconnected vortex loops similar in shape to those observed in experiments.[3,4,12] The most striking feature observed in Fig. 1b is the apparent symmetry of the wake about a plane passing through the wake centerline. Planar symmetry in the unsteady sphere wake has been observed in experiments in the range $300 < Re_d < 420$ by Sakamoto and Haniu[4] and was also observed in the simulation of Tomboulides et al.[7] at $Re_d = 300$. Although these previous as well as current flow visualizations indicate planar symmetry, the extent to which this condition is maintained in space and time has not been established, and this is the first issue addressed here.

Because vortex shedding from the sphere is a nonaxisymmetric phenomenon, it results in a side force that is analogous to the lift force on a cylinder. The actual direction of the side force is determined by the azimuthal angle where the vortex loops are formed in the near wake. Consequently, the direction of the side force can be taken as a direct indication of the azimuthal angle of vortex formation. The time variation of the drag and side force coefficient, along with the angle of the side force (in radians), is plotted in Fig. 2. Note that the actual angle does not have any significance because it depends on the initial disturbance and the subsequent evolution of the flow to the stationary state. Here the mean angle of the side force has been subtracted, and furthermore, the angle has been amplified a hundred times to show its variation clearly. The variation in the side-force angle is $\mathcal{O}(10^{-4})$, which is within the numerical uncertainty of the solver. This indicates that the vortices are formed at exactly the same location in every shedding cycle. The plot also shows that, in contrast to the cylinder wake where the maximum lift force is of magnitude comparable to the drag force,[13] the side force on the sphere is about an order of magnitude smaller than the drag force. Another difference is that, unlike the drag force on a cylinder that oscillates at twice the shedding frequency, the drag force on the sphere oscillates at the shedding frequency because in the cylinder wake two counter-rotating vortices are shed per shedding cycle, whereas in the sphere wake only one vortex loop is shed per cycle.



a)

b)

Fig. 1 Two views of the vortical structures observed in the wake at Re_d = 350. The vortical structures have been visualized by plotting λ_i = 0.1 isosurface. Note that x is the streamwise direction. a) Perspective view of the wake that shows the interconnected loop structure of the vortices. b) View along the plane of symmetry. The symmetry plane has been indicated by a dash–dot line. The isosurface has been terminated near the sphere.

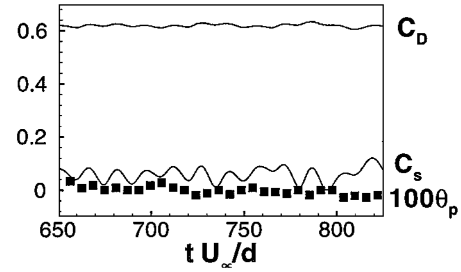


Fig. 2 Time variation of drag coefficient C_D, side-force coefficient C_s, and side-force angle θ_p for Re_d = 350.

Maintenance of planar symmetry requires that the vortices form at the same azimuthal location in every shedding cycle and that the subsequent downstream evolution of the vortices not be susceptible to nonsymmetric disturbances. Thus it is relevant to evaluate the degree to which the plane of symmetry is maintained throughout the wake. Because the azimuthal (or swirl) component of velocity is zero at the plane of symmetry, the exact orientation of the symmetry plane can be determined by computing the azimuthal angle of the zero crossing of the azimuthal velocity. At each streamwise location in the wake, the orientation of the plane of symmetry thus has been determined at a number of cross-stream locations and a mean orientation angle computed by averaging over all cross-stream locations. The three-dimensional velocity field has been extracted at 20 different time instants over three successive shedding cycles, and the preceding process of computing the orientation angle of the symmetry plane is carried out for all of these time instants. The overall variation in the mean orientation angle of the plane of symmetry over all of these time instants is $\mathcal{O}(10^{-4})$, which can be considered numerically insignificant. Thus, at this Reynolds number, the condition of planar symmetry is maintained strictly throughout the wake at all times.

The experimental visualizations of Sakamoto and Haniu[4] indicate that planar symmetry is lost at a Reynolds number of about 420, and this loss in planar symmetry is accompanied by large-scale cycle-to-cycle variations in the azimuthal orientation of the vortex loops that are formed in the near wake. However, these visualizations are mainly qualitative in nature and are not capable of identifying the presence of small-scale variations from planar symmetry that might appear at lower Reynolds numbers. Tomboulides et al.[7] also noted the absence of planar symmetry in their simulation at $Re_d = 500$. However, no attempt was made in their study to pinpoint the Reynolds number at which planar symmetry was lost. Performing a linear stability analysis to determine the Reynolds number at which this bifurcation takes place is also a nontrivial proposition because the base state is not only unsteady but also nonaxisymmetric. Thus, direct numerical simulation represents the most viable means of obtaining a better understanding of this bifurcation process.

Here we have carried out simulations at $Re_d = 375$, 400, and 425 to gain a better understanding of the bifurcation that results in the loss of planar symmetry. Each of these simulations has been continued to a stationary state, and the degree to which planar symmetry is maintained is determined through flow visualization as well as by analyzing the variation of the side-force components. Numerical calculation of the side-force angle from these two components can be difficult if the total side force approaches a small magnitude, and this is indeed what we find for these three simulations. However, the phase relationship between the two components can still be used to analyze the degree of planar symmetry in the wake. In Fig. 3 we have constructed a phase portrait by plotting the two side-force coefficients against each other for each of the four simulations, and a number of interesting observations can be made. As expected, Fig. 3a shows that the side-force components maintain a constant phase difference for $Re_d = 350$, indicating that the side-force angle does not change with time. In contrast, we find that this phase relationship is lost for $Re_d = 375$ because the phase trajectory no longer lies on a straight line, which indicates that the wake loses planar symmetry at a Reynolds number between 350 and 375. This result is not in line with experimental observations[4] that suggest loss of planar symmetry at $Re_d = 420$. Figures 3c and 3d also show that, as the Reynolds number is increased, the two force components become increasingly uncorrelated, indicating complete loss of planar symmetry at $Re_d = 425$. Finally, Figs. 3b, 3c, and 3d do not exhibit a limit-cycle-type behavior, which would be suggestive of a periodic variation in the vortex formation angle; rather, these phase portraits indicate a highly complex and nonperiodic vortex formation process.

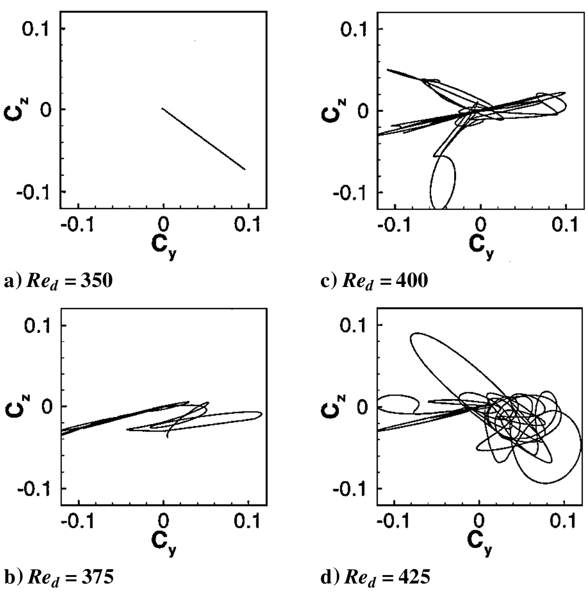


Fig. 3 Phase-plane portrait with the two side-force components plotted against each other over a time interval corresponding to several shedding cycles.

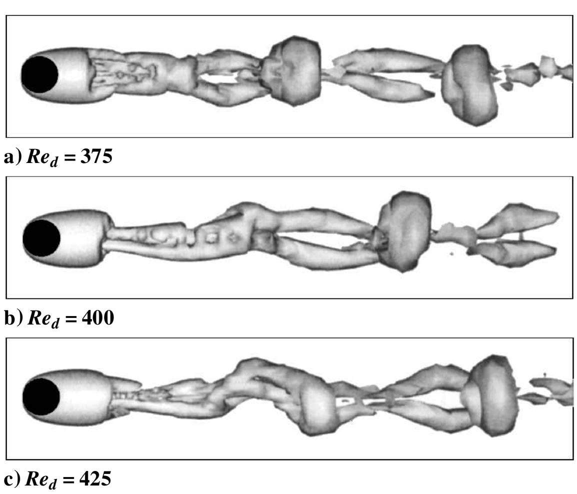


Fig. 4 Vortical structures observed in the wake at higher Reynolds numbers.

Finally, in Fig. 4 we have plotted visualizations of the vortical structures for each of these simulations, and as expected, no plane of symmetry can be observed. In addition, the visualizations indicate that at $Re_d = 375$ and 400, even though planar symmetry is absent, the cycle-to-cycle variations in the azimuthal angle of vortex formation are relatively small. In contrast, at $Re_d = 425$ we observe large cycle-to-cycle variations in the vortex formation angle, and this results in a twisting of the vortex street similar to that observed by Sakamoto and Haniu[4] for $Re_d > 420$. The fact that experiments[4] indicate loss of planar symmetry only at $Re_d = 420$ is probably because the small-scale cycle-to-cycle variations in the vortex shedding angle that appear at lower Reynolds numbers are difficult to observe through experimental flow visualizations.

Conclusions

Direct numerical simulations of uniform flow past a sphere have been used to establish that there is a regime where a strict condition of planar symmetry is maintained in the vortex shedding process. In this regime, vortex loops are formed periodically in the near wake at exactly the same azimuthal location, and furthermore, these vortex loops maintain their orientation as they convect and evolve downstream. A series of simulations in the range $350 < Re_d < 425$ indicate that the upper extent of this planar symmetric regime lies somewhere between $Re_d = 350$ and 375. This result is not in line with experimental observations[4] that indicate loss of planar symmetry at $Re = 420$. Loss of planar symmetry at these lower Reynolds numbers is accompanied by small-scale variations in the azimuthal angle of vortex formation, which could be overlooked in smoke or dye visualizations. However, at higher Reynolds numbers there are large cycle-to-cycle variations in the vortex formation angle, and these variations result in an extremely complex wake topology and vortex shedding process.

Acknowledgment

These simulations were performed on the Cray T-90 at the San Diego Supercomputer Center.

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